Electroweak Naturalness, The Little Hierarchy Problem and Natural SUSY

Lecture at the Center for Future High Energy Physics

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1 Introduction

Natural supersymmetry is a mechanism proposed to solve the *little hierarchy problem*, a problem due to the lack of an observation of *any* new physics which solves the *big hierarchy problem* in any experiment performed thus far. The little hierarchy problem has only been made worse by a lack of BSM physics results from the LHC. We will begin today by talking about the *(big) hierarchy problem*, related to the cancellation of various contributions to the Higgs mass parameter which come from energy scales of very different orders of magnitude. There are several popular resolutions of the hierarchy problem, including supersymmetry and warped extra dimensions (dual by AdS/CFT to the composite Higgs models discussed by Professor Sundrum). We will not discuss these topics much in this lecture as they've been discussed previously.

Afterwards, we'll move on to the focus of today's talk; we will discuss how natural SUSY solves the little hierarchy problem by stabilizing contributions to the Higgs mass parameter. We will discuss the interpretation of "more minimal SUSY" as "natural SUSY", the *minimal* amount of supersymmetric physics required to stabilize the little hierarchy. We will take a bottom-up approach to constructing natural SUSY and discuss constraints on the framework.

This lecture constructs a model of natural SUSY which had not been excluded in 2011 at the time of the writing of the work this lecture is based on, but with the 8 TeV data from the LHC, this particular model's fate is looking rather grim. There are slight variants of this model which are not completely ruled out yet, in particular models with baryon-number violating R-parity violation. New search strategies capable of probing this scenario are continually being developed and utilized by the CMS and ATLAS collaborations. That said, in this lecture we continue to develop natural SUSY from a historical perspective for several reasons:

- The construction of natural SUSY as the minimal amount of new physics which solves a problem represents an important tool in the methodology of a theorist, one which may well be applied to other problems that arise in the future
- We may yet discover natural SUSY "right around the corner" at the 13 and 14 TeV LHC. If so, the electroweak scale may be a little tuned, but it would offer a fantastic motiviation to pursue a higher-energy machine to discover those states which solve the big hierarchy problem
- We will explicitly see along the way the power of supersymmetry at cancelling quadratic divergences and constraining model-building

The contents of this lecture borrow heavily from the author's work in [1], and the research was performed in collaboration with Raman Sundrum, Andrey Katz and Scott Lawrence. Several other groups pursued similar lines of inquiry with partially overlapping results, including but not limited to [2, 3, 4].

2 The Big and Little Hierarchy Problems

The SM should be viewed as an effective field theory below some high-energy cutoff Λ, the scale of new physics. As an absolute maximum, this scale should be $M_{pl} \approx 1.2 \cdot 10^{19}$ GeV, the scale of quantum gravity. If Λ is up at the GUT or Planck scale, then there exists a large hierarchy between Λ and the electroweak scale v. This scale is unstable against radiative corrections coming from the standard model particles. For example, the top loop shown in fig. 1 induces a quadratically divergent contribution to the Higgs mass parameter:

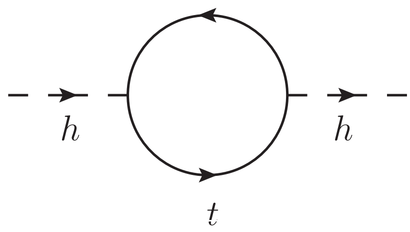

Figure 1: The SM top-loop contribution to the Higgs mass.

$$
\begin{aligned}
\delta m_h^2 &= i\,(-iy_t)^2\,(-1)N_c \int \not{d}^4 l \frac{\mathrm{Tr}(i(\not{l})i(\not{l}))}{(l^2+i\varepsilon)^2} \\
&= -\frac{6y_t^2}{16\pi^2}\Lambda^2
\end{aligned}
\tag{1}
$$

Note that this computation was performed pre-electroweak symmetry breaking (EWSB), and that we work with two-component spinor notation, not four, as the SM is chiral. In the low-energy theory, one should introduce a counterterm $V \supset -\delta_{c.t}|\phi|^2$ such that $m_h^2 = m_{h,0}^2 - \delta_{c.t.}$ to parameterize the effects of UV physics that have been integrated out. Therefore, $\delta_{c.t.}$ and therefore m_h^2 could naturally be $O(\Lambda^2)$. However, to get the correct physical Higgs mass, one must cancel the correction of all SM loops nearly entirely (but not completely) against the counterterm. In other words, it must be that m_h is related to Λ by

$$
m_{h,ren}^2 = (89\ \mathrm{GeV})^2 = m_h^2 - \frac{6}{16\pi^2} y_t^2 \Lambda^2 + \ldots \tag{2}
$$

where Λ may be as large as 10^{19} GeV. (89 GeV appears here to obtain the correct *physical* Higgs mass after EWSB, in a certain coefficient convention for the Higgs potential.) This implies that one must "fine-tune" the UV-sensitive mass parameter against quadratic corrections arising from IR physics. Strictly speaking, this is not an experimental problem, but it is quite distasteful, and we have no reason to suspect that nature should have chosen such a particular value for the bare parameter. So this issue is dubbed the "hierarchy problem"; it is occasionally called the big hierarchy problem to distinguish it from the little hierarchy problem, to be discussed below.

The hierarchy problem is considered appalling in light of the postulate of *naturalness*, which asserts that the size of quantum corrections should not be larger than the physical values of the parameters; in other words, $|\frac{\delta m_h^2}{m_{h,phys}^2}| \lesssim 1$. One should not view naturalness as a *requirement* of a theory, but rather as a postulate to be tested, as it has frequently been a good guiding principle in the past. Indeed, credence is lent to the idea of the hierarchy problem being taken seriously by previous observations of new physics stepping in to resolve other hierarchy problems. There are many proposed non-tuned resolutions of the hierarchy problem, *all* of which involve introducing new physics at the TeV-scale in order to remove quadratic sensitivity to scales above the TeV-scale. We will not go into details on these subjects as they have been covered by previous lecturers. For example, in the composite Higgs scenario discussed on Tuesday by Professor Sundrum, these new states are the hadrons of the new strongly-coupled group.

In the past few years, we have been collecting evidence that if one believes firmly that the hierarchy problem has a solution, then there is in fact a "little hierarchy problem" (LHP). This problem is essentially the non-observation of any evidence that the hierarchy problem has indeed been solved by nature. In order to eliminate fine-tuning, one generically expects new physics to have shown up by $O(100\text{ GeV})$, and certainly no later than $O(1\text{ TeV})$. However, despite the exhaustive set of experimental searches that have been performed, we have yet to find compelling evidence for the existence of new particles which solve the hierarchy problem. We have probed the SM on many frontiers; we have tested its CP physics, its flavor physics; we've performed electroweak precision measurements and performed direct searches at colliders, and yet none of these have turned up any evidence for new physics.

One might suppose that the new physics is right around the corner ($O(10\text{ TeV})$), but pushing the SUSY-breaking scale or the compositeness scale reintroduces fine-tuning at the percent-level or worse. The LHP is shared by all solutions to the hierarchy problem, and none offer resolutions of the LHP by themselves. Various solutions to the LHP exist; Twin Higgs, Little Higgs and superlatives and natural SUSY are a few examples of solutions. We will focus today on natural SUSY; in this context, the solution is the following: take the *minimal* amount of supersymmetric physics required to stabilize the little hierarchy below 10 TeV, and then push all of the *other* states up to higher energies. This strategy crucially allows for the little hierarchy to be stabilized, while removing various experimental constraints on supersymmetric models, as has been discussed extensively in the literature. However, within this framework there are still various nontrivial opportunities to discover even this minimal set of new physics. We will touch on a few such points at the end of this lecture.

In this lecture, we will explore phenomenological consequences and self-consistency of natural SUSY as a solution to the LHP. There is no reason a priori to suspect that perturbative SUSY must be *the* solution to the *big* hierarchy problem, and so we work in a bottom-up framework in our study of natural SUSY, asking what physics we absolutely *need* for the stabilization of the little hierarchy.

3 Natural SUSY

We now move on to discuss the mechanism of natural SUSY, and its implications for the little hierarchy problem. Our goal is to work from the bottom up; impose a moderate cutoff of 10 TeV on the theory, and ask what *minimal* amount of supersymmetric physics is *needed* to ensure radiative stability of the theory up to the cutoff. This constitutes a resolution to the little hierarchy problem as the minimal amount of new physics, lacking as spectacular of signatures as other points in MSSM parameter space, can elude searches at the LHC and in other experiments. Ref. [5] dubbed this kind of structure "effective SUSY"; in this lecture we will refer to it as "natural SUSY", as we shall explain later. Since [6, 5], a number of quite different approaches to far-UV dynamics have converged on such a "more minimal" spectrum at accessible energies [7, 8, 9, 10]. The significance of the 10 TeV scale is that almost all experiments, up to and including the LHC, only have sensitivity to new physics $\lesssim$ 10 TeV, be it through direct searches or virtual effects. In this regard, flavor physics tests are exceptional in probing vastly higher scales and consequently they require special consideration.

What we will find, as mentioned in [11, 12] and developed in [6], is that superpartners with $O(1)$ coupling to the SM are required for naturalness up to 10 TeV. In natural SUSY, these superpartners are the stops, sbottoms, gauginos and the Higgs sector, but with other squarks and sleptons heavy and beyond reach of the LHC. This particle content satisfies the criterion of being natural up to 10 TeV. The omitted superpartners may play a crucial role in weak scale stability up to much higher scales, but this is outside the scope of the effective theory and outside the grasp of the LHC. We will investigate the range of squark, gaugino and Higgsino masses for which the electroweak scale is natural, as well as explore indirect constraints on this scenario, in particular from flavor tests. A more thorough treatment of this scenario can be found in [1].

Naturalness is our driving concern when building our BSM framework. However, one must generally weigh naturalness against a number of concerns of the SUSY paradigm which at least partially relate to very high energies. Today, however, we study minimal effective theories that arise from insisting on naturalness of the low-energy theory. They are "minimal" in terms of the particle content and parameter space of $\mathcal{L}_{eff}$. This does *not* imply, however, that their UV-completions are also minimal in some way. Conversely, the MSSM is a minimal visible sector from the high-energy perspective, but is non-minimal in the sense that matters to the LHC effective theory and phenomenology.

Of course, there is no guarantee that at accessible energies new SUSY physics will be turn out to be minimal. Rather, we study minimal LHC-effective theories for three reasons:

- They represent possible SUSY phenomenology, and there do exist UV SUSY dynamics that match onto them
- A part of the natural parameter space remains open, and yet is discoverable by the LHC
- Minimal models in any arena of exploration represent an important departure point for thinking more broadly.

Today, we will take a relatively UV-agnostic approach to the minimal effective theory and see what implications it has for phenomenology.

3.1 Field Content

Let us start with the MSSM field content and ask which superpartners are minimally needed in order to maintain electroweak naturalness below 10 TeV, roughly the collider reach in the years to come. We will not ask here what physics lies above this scale, though this is an exciting question with many possible implications for a 100 TeV machine. Today, though, $\Lambda \equiv 10$ TeV provides the cutoff for any UV divergences encountered in the effective theory, and this allows us to estimate electroweak fine-tuning and check where in parameter space natural SUSY solves the little hierarchy problem of the SM.

Before we begin, though, we state the findings of this study so as to lay out conventions and define the Lagrangian that we use. SM particles with order one couplings to the Higgs boson must certainly have superpartners in the effective theory because they would otherwise give rise to quadratically divergent Higgs mass-squared contributions at one loop, $\sim \Lambda^2/(16\pi^2)$, big enough to require significant fine-tuning. Those order-one parameters are the top Yukawa coupling y_t, the three gauge couplings, and the Higgs quartic. In order to supersymmetrically cancel these divergences, the effective theory must therefore include the left-handed top and bottom squarks, $\tilde{q}_L \equiv (\tilde{t}_L, \tilde{b}_L)$, and the right-handed top squark, $\tilde{t}_R$, as well as the up-type Higgsino, $\tilde{h}_u$, and electroweak gauginos, $\lambda_{1,2}$. In addition, we will argue that we need the down-type Higgs h_d and Higgsino $\tilde{h}_d$ as well as the right-handed sbottom squark $\tilde{b}_R$ and the gluino λ_3.

Therefore, the effective theory has the following complete supermultiplets:

$$
\begin{aligned}
Q \equiv \begin{pmatrix} T \\ B \end{pmatrix} \equiv (\tilde{q}_L, q_L) \equiv \left(\begin{pmatrix} \tilde{t}_L \\ \tilde{b}_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \right) \\
\bar{T} \equiv (\tilde{t}^c_R, t^c_R) \\
\bar{B} \equiv (\tilde{b}^c_R, b^c_R) \\
H_u \equiv (h_u, \tilde{h}_u) \\
H_d \equiv (h_d, \tilde{h}_d) \\
V_1 \equiv (B_\mu, \lambda_1) \\
V_2 \equiv (W_\mu, \lambda_2) \\
V_3 \equiv (G_\mu, \lambda_3)
\end{aligned} \tag{3}
$$

3.2 Effective Lagrangian, Neglecting Third-Generation Mixing

Above, we have introduced squarks belonging to only the "third generation", and yet this notion is slightly ambiguous because generation-numbers are not conserved, even in the SM. However, CKM mixing involving the third generation is at least highly suppressed, so we will begin by considering the "zeroth order" approximation in which third-generation number is exactly conserved. For most purposes in LHC studies of the new physics, this approximation is sufficient. But for complete realism and to check the viability of the theory in the face of very sensitive low-energy flavor constraints, the extra subtlety of third-generation mixing must be taken into account. We defer this discussion until section 5. For now, this mixing is formally "switched off". Further, we will impose R-parity on natural SUSY, though the R-parity violating case is both theoretically motivated as well as being phenomenologically interesting.

With the field content described above, the effective Lagrangian is given by

$$\begin{aligned}
\mathcal{L}_{eff} \;=\; & \int d^4\theta K + \left(\int d^2\theta \left(\frac{1}{4}\mathcal{W}_\alpha^2 + y_t \bar{T} H_u Q + y_b \bar{B} H_d Q + \mu H_u H_d \right) + \text{h.c.} \right) \\
& + \mathcal{L}_{kin}^{light} - \left(\bar{u}_R Y_u^{light} h_u \psi_L + \bar{d}_R Y_d^{light} h_d \psi_L + \text{h.c.} \right) + \mathcal{L}_{lepton} \\
& - m_{\tilde{q}_L}^2 |\tilde{q}_L|^2 - m_{\tilde{t}_R^c}^2 |\tilde{t}_R^c|^2 - m_{\tilde{b}_R^c}^2 |\tilde{b}_R^c|^2 - m_{h_u}^2 |h_u|^2 - m_{h_d}^2 |h_d|^2 \\
& - \left(m_{i=1,2,3} \lambda_i \lambda_i + B\mu h_u h_d + A_t \tilde{t}_R^c h_u \tilde{q}_L + A_b \tilde{b}_R^c h_d \tilde{q}_L + \text{h.c.} \right) \\
& + \mathcal{L}_{hard} + \mathcal{L}_{non-ren.} \qquad (4)
\end{aligned}$$

where the first line is in superspace/superfield notation, while the remaining lines are in components. Here, K is the standard gauge-invariant Kähler potential for the chiral superfields of Eq. (3), and $\mathcal{L}_{kin}^{light}$ denotes the standard gauge-invariant kinetic terms for the light SM quarks (that is, not the top and bottom), $u_R, d_R, \psi_L \equiv (u_L, d_L)$. $\mathcal{L}_{lepton}$ denotes all terms involving leptons, with Yukawa couplings to h_d (neglecting neutrino mass terms). The super-field strength tensors are implicitly summed over all three gauge groups of the standard model. Even the second line can be thought of as the result of starting from the supersymmetric MSSM, but then deleting all superpartners for light SM fermions. As mentioned above, we ignore the third generation mixing with the first two generations (until section 5). The third and fourth lines are soft SUSY breaking terms for the superfields of the effective theory. The fifth line is not relevant for the purposes of this talk; for more details, we refer the reader to [1].

4 Naturalness Constraints

Here, we assemble the electroweak naturalness constraints on natural SUSY, thereby giving a rough idea of the motivated regions of its parameter space. For this purpose, we will compute various independent corrections to the h_u mass-squared, and simply ask them to be $\lesssim$ (200 GeV)^2 for naturalness. We will compute these corrections *before* EWSB. Contributions sensitive to EWSB are typically $\sim O((100\text{ GeV})^2)$, and therefore typically do not compromise naturalness. Given the intrinsically crude nature of naturalness arguments, we see no merit in a more refined analysis.

We begin with a classical "tuning" issue. The μ term gives a supersymmetric $|\mu|^2$ contribution to the Higgs mass-squareds. While the soft terms also contribute to Higgs mass-squareds, naturalness forbids any fine cancellations, so therefore by the criterion stated above,

$$|\mu| \lesssim 200\text{ GeV} \tag{5}$$

This same parameter then also plays the role of the Higgsino mass parameter, ensuring[1] relatively light charginos and neutralinos in the superpartner spectrum. (Of course, after EWSB, these physical states may also contain admixtures of electroweak gauginos.)

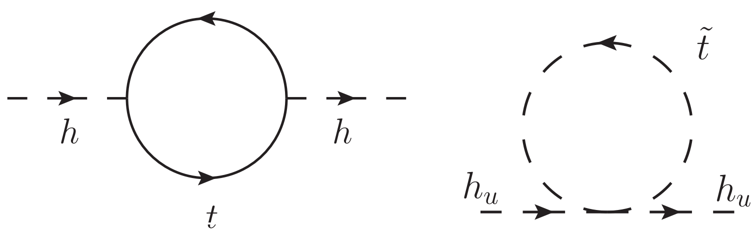

Figure 2: Higgs mass corrections

Next, we turn to quantum loops. We assume that $\tilde{q}_L, \tilde{t}_R$ have approximately the same mass, $m_{\tilde{t}}$, for simplicity, and we also neglect the μ and A-terms. We work pre-EWSB since we are concerned with sensitivity to parametrically higher scales. We now evaluate the diagrams in figure 2. Note that we ignore finite terms, assume the stops are the same mass for simplicity and assume $m_{\tilde{t}} \ll \Lambda$. Finally, the resulting integrals can be performed with identities given in the appendix.

$$\begin{aligned}\delta m_{h,t}^2 &= i\,(-iy_t)^2\,(-1)N_c \int \bar{d}^4 l \frac{\mathrm{Tr}(i(\not{l})i(\not{l}))}{(l^2+i\varepsilon)^2} \\ &= -\frac{6y_t^2}{16\pi^2}\Lambda^2\end{aligned} \tag{6}$$

[1]This argument is not infallible; see [1] for more details.

$$
\begin{aligned}
\delta m^2_{h,\tilde{t}} &= i(2)(-iy_t^2)N_c \int d\!\!{}^-{}^4 l \frac{i}{l^2 - m^2_{\tilde{t}} + i\varepsilon} \\
&= \frac{6y_t^2}{16\pi^2}\left(\Lambda^2 - 2m^2_{\tilde{t}} \ln\left(\frac{\Lambda}{m_{\tilde{t}}}\right)\right)
\end{aligned}
\tag{7}
$$

Summing these, we find that the $m^2_{h_u}$ parameter receives the following correction:

$$
\delta m^2_{h_u} = -\frac{3y_t^2}{4\pi^2} m^2_{\tilde{t}} \ln\left(\frac{\Lambda}{m_{\tilde{t}}}\right) \tag{8}
$$

Naturalness therefore requires, very roughly,

$$
m_{\tilde{t}} \lesssim 400\text{GeV} \tag{9}
$$

Note that this example illustrates a general theme in naturalness considerations; we have added in a new state which "cuts off" the quadratic divergence and replaces it with a quadratic sensitivity to the mass of a new physics state (in this case $m_{\tilde{t}}$). In other resolutions of both the little and big hierarchy problem, the same mechanism of cutting off divergences takes place.

There are also electroweak gauge/gaugino/Higgsino one-loop contributions to Higgs mass-squared. Again, working before electroweak symmetry breaking (gaugino-Higgsino mixing) and just looking at the stronger $SU(2)_L$ coupling, the Higgs self-energy diagrams are in figure 3. There are nontrivial gauge index contractions in these diagrams; in all four, they contract to return the quadradic Casimir C_2.

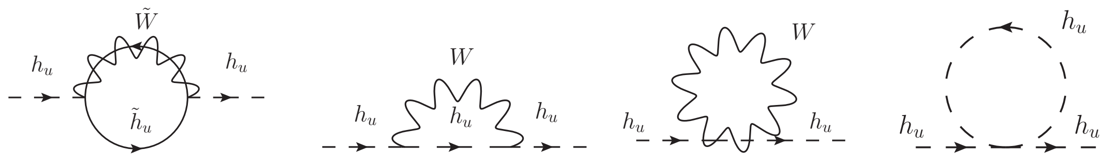

Figure 3: Higgs mass correction

$$
\begin{aligned}
\delta m^2_{h,\tilde{h}-\tilde{W}} &= i(-i\sqrt{2}g_2)^2 C_2(-1) \int d\!\!{}^-{}^4 l \,\mathrm{Tr}\left(\frac{\not{l}}{l^2 - m^2_{\tilde{h}} + i\varepsilon}\frac{\not{l}}{l^2 - m^2_{\tilde{W}} + i\varepsilon}\right) \\
&= -\frac{4g_2^2 C_2}{16\pi^2}\left(\Lambda^2 - 2(m^2_{\tilde{h}} + m^2_{\tilde{W}}) \ln\left(\frac{\Lambda}{m_{\tilde{W}}}\right)\right)
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
\delta m^2_{h,h-W} &= i(-ig_2)^2 C_2 \int d\!\!{}^-\!{}^4 l \; l^\mu \frac{-i\eta_{\mu\nu}}{l^2+i\varepsilon} l^\nu \frac{i}{l^2+i\varepsilon} \\
&= -\frac{g_2^2 C_2}{16\pi^2}\Lambda^2
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
\delta m^2_{h,W} &= i(2ig_2^2)\frac{1}{2}C_2 \int d\!\!{}^-\!{}^4 l \frac{-i\delta^\mu_\mu}{l^2+i\varepsilon} \\
&= \frac{4g_2^2 C_2}{16\pi^2}\Lambda^2
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
\delta m^2_{h,h} &= i(-ig_2^2)C_2 \int d\!\!{}^-\!{}^4 l \frac{i}{l^2+i\varepsilon} \\
&= \frac{g_2^2 C_2}{16\pi^2}\Lambda^2
\end{aligned}
\tag{13}
$$

For $SU(2)$, the quadratic Casimir is $C_2 = \frac{3}{4}$, and so the Higgs mass correction is then given by

$$
\delta m^2_{h_u} = \frac{3g_2^2}{8\pi^2}(m^2_{\tilde{W}} + m^2_{\tilde{h}}) \ln \frac{\Lambda}{m_{\tilde{W}}}
\tag{14}
$$

We identify the Higgsino mass with μ. Because we are already taking $\mu \lesssim 200$ GeV, this translates into a roughly natural wino mass range of

$$
m_{\tilde{W}} \lesssim \text{TeV}
\tag{15}
$$

Considerations beyond SUSY itself imply that we need to retain even more superpartners. Electroweak gauge anomaly cancellation implies that $\tilde{h}_u$ must be accompanied by $\tilde{h}_d$ in the effective theory. Indeed, one might have anticipated that down-type Higgs *bosons*, h_d, are required anyway to give masses to the down-type fermions, and that $\tilde{h}_d$ provide the required superpartners.[2]

There are quartic scalar terms in the Lagrangian that would give rise to quadratic contributions

[2]We proceed with this logic in this section, although there is a loop-hole whereby h_u can provide down-type fermion masses in the effective theory, and h_d bosons are not needed. However, we will not discuss this option further here.

to the Higgs mass, arising from the D-terms of the various groups. In general, though, they are proportional to Tr T^a and therefore vanish for $SU(N)$. However, the same statement is not true for $U(1)$ gauge groups. We are therefore led to consider the contributions from $U(1)_Y$. It is associated by supersymmetry with the mixed hypercharge-gravity triangle anomaly. We compute the hypercharge D-term loop contribution to Higgs mass-squared, in figure 4:

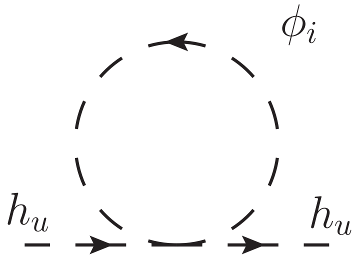

Figure 4: Higgs mass correction

This gives rise to a higgs mass correction:

$$\begin{aligned} \delta m_{h_u}^2 &= i \sum_{\text{scalars } j} (-iY_{h_u} Y_j) \int d\!\!{}^-{}^4 l \frac{i}{l^2 - m_j^2 + i\varepsilon} \\ &= \sum_{\text{scalars } j} \frac{{g_2}^2 Y_j Y_{h_u}}{16\pi^2} \left(\Lambda^2 - m_j^2 \ln \frac{\Lambda^2 + m_j^2}{m_j^2} \right) \end{aligned} \tag{16}$$

Including both the right-handed sbottom and the down-type higgs, as we do in this section, ensures that the quadratic divergence cancels, but there is still a residual correction to the Higgs mass. Given that other scalars have already been argued to be relatively light, we can use this correction to estimate the natural range for the mass of $\tilde{b}_R$,

$$m_{\tilde{b}_R} \lesssim 3\text{TeV} \tag{17}$$

For the most part, two-loop quadratic divergences $\sim \Lambda_{UV}^2/(16\pi^2)^2$ are not important for Higgs naturalness, with a cutoff as low as 10 TeV. But the QCD coupling is an exception. In particular, the $\tilde{q}_L, \tilde{t}_R^c$ masses must themselves be so light in order to protect Higgs naturalness at one loop order, that they suffer from their own naturalness problem due to one-loop mass corrections from QCD. This one loop QCD destabilization of the squarks, hence two-loop destabilization of the

Higgs, requires the gluino, λ_3, to be in the effective theory. We compute those mass corrections, which are dominated by the diagrams in figure 5:

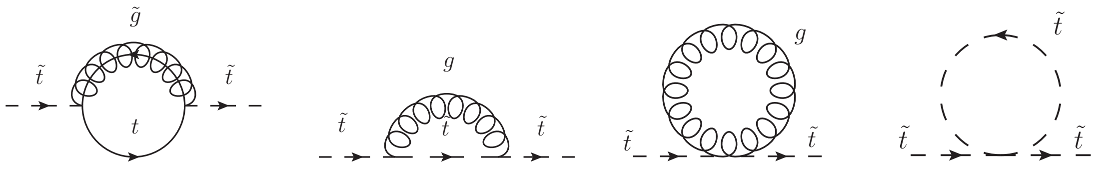

Figure 5: Stop mass correction

$$\begin{aligned}\delta m^2_{\tilde{t},t-\tilde{g}} &= i(-i\sqrt{2}g_3)^2 C_2(-1)\int d\!\!\!{}^-{}^4 l \,\mathrm{Tr}\left(\frac{\not{l}}{l^2 - m^2_{\tilde{g}} + i\varepsilon}\frac{\not{l}}{l^2 + i\varepsilon}\right)\\ &= -\frac{4g_3^2 C_2}{16\pi^2}\left(\Lambda^2 - 2m^2_{\tilde{g}}\ln\left(\frac{\Lambda}{m_{\tilde{g}}}\right)\right)\end{aligned} \tag{18}$$

$$\begin{aligned}\delta m^2_{\tilde{t},\tilde{t}-g} &= i(-ig_3)^2 C_2 \int d\!\!\!{}^-{}^4 l \; l^\mu \frac{-i\eta_{\mu\nu}}{l^2 + i\varepsilon} l^\nu \frac{i}{l^2 - m^2_{\tilde{t}} + i\varepsilon}\\ &= -\frac{g_3^2 C_2}{16\pi^2}\left(\Lambda^2 - 2m^2_{\tilde{t}}\ln\left(\frac{\Lambda}{m_{\tilde{t}}}\right)\right)\end{aligned} \tag{19}$$

$$\begin{aligned}\delta m^2_{\tilde{t},g} &= i(2ig_3^2)\frac{1}{2}C_2 \int d\!\!\!{}^-{}^4 l \frac{-i\delta^\mu_\mu}{l^2 + i\varepsilon}\\ &= \frac{4g_3^2 C_2}{16\pi^2}\Lambda^2\end{aligned} \tag{20}$$

$$\begin{aligned}\delta m^2_{\tilde{t},\tilde{t}} &= i(-ig_3^2)C_2 \int d\!\!\!{}^-{}^4 l \frac{i}{l^2 - m^2_{\tilde{t}} + i\varepsilon}\\ &= \frac{g_3^2 C_2}{16\pi^2}\left(\Lambda^2 - 2m^2_{\tilde{t}}\ln\left(\frac{\Lambda}{m_{\tilde{t}}}\right)\right)\end{aligned} \tag{21}$$

The quadratic Casimir for $SU(3)$ is $C_2 = \frac{4}{3}$, giving rise to a stop mass correction:

$$\delta m_{\tilde{t}}^2 = \frac{2g_3^2}{3\pi^2} m_{\tilde{g}}^2 \ln \frac{\Lambda}{m_{\tilde{g}}} \tag{22}$$

For squark masses $\sim$ few hundred GeV, naturalness requires

$$m_{\tilde{g}} \lesssim 2m_{\tilde{t}} \tag{23}$$

5 Flavor-Changing Neutral Currents and CP Violation

Above, we have worked in the drastic approximation that the mixing between the third generation with the first two generations vanishes, so that the meaning of "third generation" squarks, $\tilde{q}_L, \tilde{t}_R^c, \tilde{b}_R^c$, is completely unambiguous. In this limit, there is a conserved third-generation (s)quark number. In the real world, third generation mixing is non-zero but small. In the Wolfenstein parametrization, mixing with the second generation is of order ϵ^2 and mixing with the first generation is of order ϵ^3, where $\epsilon \sim 0.22$ corresponds to Cabibbo mixing. Given this fact, it is more natural to have comparable levels of violation of third-generation (s)quark number in the physics we have added beyond the SM.

In practice this means that for every interaction term in which the squarks currently appear, where third-generation number is conserved by the presence of t or b quarks (in electroweak gauge basis), we now allow more general couplings, with the third generation quarks replaced by quarks of the first and second generations. The associated couplings with second generation quarks are taken to be of order ϵ^2, while those with first generation quarks are taken to be of order ϵ^3, all in electroweak gauge basis. All these couplings involving the squarks are technically hard breaking of SUSY, but $\epsilon^{2(3)}$ is so small that, like other hard breaking in the effective theory, they do not spoil Higgs naturalness below 10 TeV. For most, but not all, of the LHC collider phenomenology the small $\epsilon^{2(3)}$ effects are negligible and we can proceed with our earlier effective Lagrangians. (We must of course keep SM third generation mixing effects, so that, for example, the bottom quark decays.) But in the more realistic setting with third-generational mixing, we must confront the SUSY flavor problem. In natural SUSY, this problem has two faces, IR and UV.

The UV face of the problem is contained in the non-renormalizable interactions of eqn. (4). For example, they can include flavor-violating interactions such as $\bar{s}d\bar{s}d$. If such a non-renormalizable interaction were suppressed only by (10 TeV)^2, it would lead to FCNCs in kaon mixing, orders of magnitude greater than observed. It is therefore vital for the non-renormalizable interactions to have a much more benign flavor structure. Whether this is the case or not is determined by matching to the full theory above 10 TeV, IR natural SUSY considerations alone cannot decide the issue. References [7, 9] are examples of UV theories which reduce to natural SUSY at accessible energies and automatically come with the kind of benign UV flavor structure we require. In this lecture, we simply assume that the UV-sensitive non-renormalizable interactions are sufficiently flavor-conserving to avoid conflict with FCNC constraints.

There remain FCNC effects that are UV-insensitive but are assembled in the IR of the effective theory through the small $\epsilon^{2(3)}$ flavor-violating couplings. Many of these have been studied in [20] and are small enough to satisfy current constraints. Indeed this feature is one of the selling points of natural SUSY. Here, we illustrate one such FCNC effective interaction for (CP-violating) $K-\bar{K}$ mixing arising as a SUSY "box" diagram. Similar processes were studied in [21, 22, 23, 24], with minor adaptations needed in our case.

While the effect is suppressed by $O(\epsilon^{10})$ in natural SUSY, it is more stringently constraining than $B_d-\bar{B}_d$ mixing or $B_s-\bar{B}_s$ mixing, even though these are suppressed by just $O(\epsilon^6)$ and $O(\epsilon^4)$ respectively. We show that with our rough flavor-changing power-counting the $\tilde{b}_R$ squark is constrained to lie above several TeV in the absence of flavor-parameter tuning.

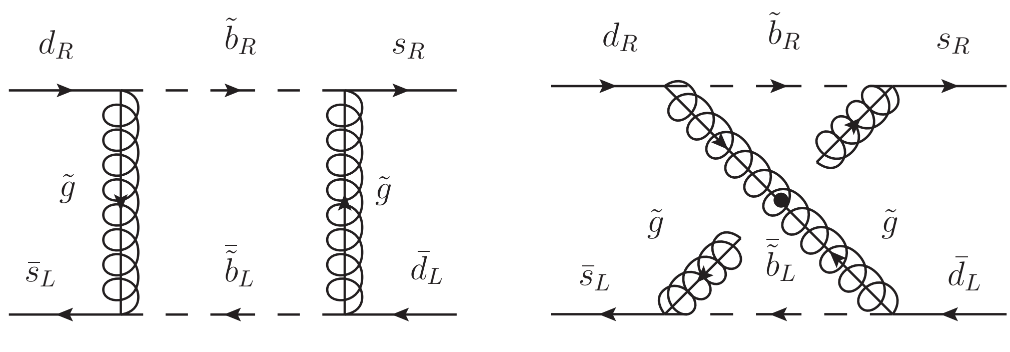

Figure 6: Contributions to $K-\bar{K}$ mixing

In a low-energy effective Lagrangian to be run down to the hadronic scale, we match onto effective operators of the form

$$\mathcal{L}_{eff} \supset \kappa(\bar{s}_L d_R)(\bar{s}_R d_L) \tag{24}$$

Strictly speaking there are two different operators depending on color contraction. As shown in [21] an operator $\mathcal{O}_5 \propto \bar{d}_R^i s_L^j \bar{d}_L^j s_R^i$ (where i, j are color indices) is not enhanced by QCD running and has $1/N_c$-suppressed QCD matrix element. Therefore we concentrate on $\mathcal{O}_4 \propto \bar{d}_R^i s_L^i \bar{d}_L^j s_R^j$, which has enhanced QCD running and large hadronic matrix element. Therefore, for the purpose of our simple estimate, in (24) we only study the case where each bilinear is a color singlet.

As discussed above, we assume that the squark couplings to second generation quarks are assigned strength $\sim g_3\epsilon^2$, while squark couplings to the first generation are $\sim g_3\epsilon^3$. We neglect $\tilde{b}_L$-$\tilde{b}_R$ mixing (after EWSB). Note that the result of evaluating these diagrams contains large logarithms of the form $\ln m_{\tilde{b}_R}^2/m_{\text{squark}}^2$, which in principle should be resummed (for example, see [24]). However, we do not do this since we only seek an estimate for κ.

Current constraints on ϵ_K require that [25]

$$(\text{Im}(\kappa)) \lesssim \left(\frac{1}{3 \times 10^5 \text{ TeV}}\right)^2 \tag{25}$$

For $m_3 \sim$ TeV and $m_{\tilde{q}_L} \sim 350$ GeV, this translates into a bound on $\tilde{b}_R$ mass of roughly $m_{\tilde{b}_R^c} \gtrsim$ 17 TeV.

Of course, this bound is extremely sensitive to our estimates for the flavor-changing vertices. For example, if each flavor-changing vertex were only half as strong as our above estimates, the bound would be relaxed to $m_{\tilde{b}_R^c} \gtrsim 4$ TeV, roughly consistent with the requirements of naturalness in section 4. Alternatively, there may be small phases present in the vertices that further suppress κ.

We list results here for some various useful scalar integrals, computed using a hard cutoff. These can be obtained from the tensor integrals in this paper by a tensor reduction procedure. These are obtained by a Wick rotation into Euclidean space, and evaluated from Euclidean momentum $0 \leq p_E^2 \leq \Lambda^2$. We neglect finite contributions as they do not matter for the purposes we need these integrals for.

$$\int \frac{đ^4 l}{l^2 - m^2 + i\varepsilon} = -\frac{i}{16\pi^2}\left(\Lambda^2 - m^2 \ln\left(\frac{\Lambda^2 + m^2}{m^2}\right)\right) \tag{26}$$

$$\int \frac{đ^4 l}{(l^2 - m^2 + i\varepsilon)(l^2 - M^2 + i\varepsilon)} = \frac{i}{16\pi^2}\left(\ln\left(\frac{\Lambda^2 + M^2}{M^2}\right) + \text{finite}\right); (M > m) \tag{27}$$

$$\int \frac{đ^4 l (l^2 + A)}{(l^2 - m^2)(l^2 - M^2)} = -\frac{i}{16\pi^2}\left(\Lambda^2 - (A + m^2 + M^2)\ln\left(\frac{\Lambda^2 + M^2}{M^2}\right) + \text{finite}\right); (M > m) \tag{28}$$

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